Estimation of model parameters in nonlocal damage theories by inverse analysis techniques

STW/PPM    DCT  55.3923

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Nieuwegein, November 9th 2005
Outline

- Setting the scene
- Project objectives
- Forward problem
- Inverse problem
- The inverse techniques
- Experimental data
- Results
- Conclusions
Setting the scene

Experimental world

Numerical world

limited amount of work

mathematical equations containing parameters

Computational output

Experimental output

concrete specimen
Setting the scene

Result:

“Quit moving!”
Setting the scene

Experimental world

Numerical world

mathematical equations containing parameters

Computational output

my PhD project

Experimental output
Parameters identification of continuum damage models:

- Study of the aspects related to the solution of the inverse problem:
  - Uniqueness and robustness of the solution (well-posedness of the inverse problem)
  - Factors of influence for the solution (e.g. experimental uncertainty, initial guess)
  - Qualitative and quantitative choice of the experimental data

- Study of the aspects related to the choice of the inverse technique (best strategy)
  - Effectiveness (how close to the solution)
  - Efficiency (time)
  - Reliability
Project objectives

- **Insight in the calibrated numerical model:**
  - solving the inverse problem *needs* insight in the forward problem, otherwise it reduces to mere data fitting
  - solving the inverse problem *helps* to have insight in the forward problem (e.g. length scale)
  - Investigation of the limitations of applicability, reliability and predictive capabilities of the calibrated numerical model (*size effect and geometry effect*)
Project objectives

- Study of the problem of objectively extracting intrinsic material properties from structural experimental responses:

  - **Numerical model is an approximation of the reality**
    - many external factors that play a role in the laboratory tests are difficult to be identified, quantified and included in the model
    - acting only on the model parameters may not be sufficient to cover the approximation
    - consequence: not constant material parameters.

  - **Possible dependency of the material parameters from**
    - structural factors: the boundary conditions, the load conditions, the specimen size and geometry
    - environmental and manufacturing factors
    - time and/or deformation state

  - **Inverse problem only valid tool to link local law at the material point level with structural response**
The numerical model (forward problem)

Gradient-enhanced continuum damage model

\[
\begin{align*}
\sigma &= (1 - \omega)D^e \varepsilon \\
\varepsilon_{eq} &= \varepsilon_{eq}(\varepsilon) \\
\omega &= \omega(\kappa) \\
\kappa &= \max(\kappa_i, \varepsilon_{eq}) \\
\varepsilon_{eq} - c \nabla^2 \varepsilon_{eq} &= \varepsilon_{eq}
\end{align*}
\]

tensile-compressive strength ratio

elastic parameters

damage threshold

Simplification:

\[
x^T = [\alpha, \beta, c]
\]
The Inverse Problem

Minimization of an objective function

\[ f = f ((y_{\text{comp}}^t(x) - y_{\text{exp}}^t), x_0) \]
The Inverse Problem

Definition of the objective function

\[ y_{\text{comp}}(x) = \begin{bmatrix} F_{\text{comp},1}(x) & \cdots & F_{\text{comp},i}(x) & \cdots & F_{\text{comp},N}(x) \end{bmatrix}^T \]

\[ y_{\text{exp}} = \begin{bmatrix} F_{\text{exp},1} & \cdots & F_{\text{exp},i} & \cdots & F_{\text{exp},N} \end{bmatrix}^T \]

\[ f(x) = (y_{\text{comp}}(x) - y_{\text{exp}})^T \cdot C_{\text{exp}}^{-1} \cdot (y_{\text{comp}}(x) - y_{\text{exp}}) = \sum_{i=1}^{N} \frac{1}{C_{\text{exp},i}^2} (F_{\text{comp},i}(x) - F_{\text{exp},i})^2 \]

weighted squared distance between exp and comp vectors
The inverse techniques

- KNN method

- Kalman filter method
K-Nearest Neighbors method (KNN)

\[ \hat{x} = \min_x f(x) \]

- choose a population of model parameters sets \( x_i \) (creation of a grid)

- compute (forward problem) \( y_{\text{comp}}(x_i) \)

- evaluation of the weighted Euclidean distance \( f(x_i) \)

- choose \( x \) that corresponds to the nearest neighbor of \( y_{\text{exp}} \) (K=1)
Kalman Filter method (KF)

Global iterations for non linear forward operator
The inverse techniques

- **KNN method**
  - Derivative free method
  - General overview in the parameters space
  - Estimation of the initial guess
  - Parallel solutions of the forward problem
  - Easily usable for any numerical model (external tool)

- **Kalman filter method**
  - Refine the searching process
  - Parameters update during fracture process
Experimental data 1

Tensile size effect tests on dog-bone shaped specimens by van Vliet and van Mier (2000)

Six specimen sizes in a scale range of 1:32
Experimental data 2

Tensile Size Effect Tests (different concrete mixes) by K. Hariri (2000)

➢ Three point bending tests (BG) on single-edge-notched concrete beams
➢ Uniaxial tensile tests (KG) on double-notched concrete prisms

<table>
<thead>
<tr>
<th>Double-edge notched tensile specimens</th>
</tr>
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<tbody>
<tr>
<td>Size</td>
</tr>
<tr>
<td>KG 1</td>
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<tr>
<td>KG 2</td>
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<tr>
<td>KG 3</td>
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<td>KG 4</td>
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<table>
<thead>
<tr>
<th>Single-edge notched bending specimens</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
</tr>
<tr>
<td>BG 1</td>
</tr>
<tr>
<td>BG 2</td>
</tr>
<tr>
<td>BG 3</td>
</tr>
<tr>
<td>BG 4</td>
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<tr>
<td>BG 5</td>
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</tbody>
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Experimental data 2

- Experimental available results:

- Speckle Interferometry for the FPZ-size evaluation
Objective of the fitting

1. Global curve one single size

2. Global + local curve one single size

3. Size effect curve (only peaks)

4. Global curves different sizes

5. Global + local curves different sizes

6. Global + local curves different sizes and geometry
Results

Global curve one single size

Ill-posed inverse problem:

- not unique parameters set
- $c$ and $\beta$ correlated
Results

2 Global + local curve one single size

\[
\begin{align*}
 f(x) &= \frac{1}{(F_{\text{comp}}(x) - F_{\text{exp}})^T (F_{\text{comp}}(x) - F_{\text{exp}}) + (\varepsilon_{\text{comp}}(x) - \varepsilon_{\text{exp}})^T (\varepsilon_{\text{comp}}(x) - \varepsilon_{\text{exp}})} \\
 &\quad \times \frac{1}{(F_{\text{exp}})^T F_{\text{exp}}} - \omega_{\text{exp}} \\
 &= 2
\end{align*}
\]
Results

✓ Single parameters set identified
✓ Fitting of other sizes curves not guaranteed
Results

③ Size effect curve (only peaks)

✓ Different parameter sets could give “good” average fitting
✓ Fitting of the entire global curves not guarantied
✓ No unique parameters set reproduces the real size effect curve (statistical effects not captured by the deterministic model)
✓ The length scale may be used as tuner parameter.
Results

4 Global curves different sizes

- Different parameter sets could give “good” average fitting.
- No unique parameters set reproduces the real size effect curve.
- The length scale may be used as tuner parameter.
Results

Global + local curves different sizes

- Single parameters set may be identified.
- No unique parameters set reproduces the real size effect curve.
- The length scale may be used as tuner parameter.
Results: no unique parameters set reproduces the real size effect curve.
Results: fitting only the peaks ≠ fitting the entire global curves
Results: the length scale may be used as tuner parameter

BG specimens

[Graph showing log $\sigma$ (MPa) vs. log $W_{eff}$ (mm) with data points and best fit curves]

KNN results (c and beta minimum)

[Graph showing $\beta$ vs. $c$ with markers for different BG specimens]

STRESS-CMOD curves (BG1)

[Graph showing stress vs. CMOD with curves for different conditions]

STRESS-CMOD curves (BG2)

[Graph showing stress vs. CMOD with curves for different conditions]

STRESS-CMOD curves (BG3)

[Graph showing stress vs. CMOD with curves for different conditions]

STRESS-CMOD curves (BG5)

[Graph showing stress vs. CMOD with curves for different conditions]
Results

- Global + local curves different sizes and geometry

- Structural effect

![Graph showing BG and KG specimens with different log Weff (mm) and log σn (MPa) data]

- BG specimens
  - exp data
  - c=20 beta=300
  - c=60 beta=300
  - c=20 beta=1100

- KG specimens
  - exp data
  - c=20 beta=300
  - c=60 beta=300
  - c=20 beta=1100
  - c=80 beta=300
Conclusions (parameters identification strategy)

Parameters identification procedure (for $\beta$ and $c$):

1. KNN method to identify the best glob for each size.
2. Find the better fixed $\beta$s (better representatives of the “best glob” population)
3. Find for each better $\beta$ the best $c$ for each size. (peaks below the computational curve of a fixed $[\beta, c]$ couple have smaller $c$ and vice versa)
4. Find the best fixed $c$ for each better $\beta$ (the best representative of each “fixed $\beta$” population)
5. Find the best $[c, \beta]$ set considering the local curves (priority of fitting to the glob curves)

How the objective has to be fitted?

- Simply looking in the KNN matrix
- Penalty of not a unique $c$
- Penalty of not fitting local data

- KF on one par set for all the sizes
- Penalty of not unique $\beta$
- KF on $c$ for each size
- Stop
- Stop
- Go back to step 6 for another $[c, \beta]$ couple
- Use the best glob of step 1
- Stop
- Use the $c$s of step 3 for that fixed $\beta$ (use $c$ as tuner parameter)
- “average” global fitting through all the sizes is satisfactory?
- “average” global fitting through all the sizes is satisfactory?
- Yes
- Yes
- No
- No
- 8 KF on one par set for all the sizes
- 8 Go back to 6 for another $[c, \beta]$ couple

BG specimens

Parameters identification procedure (for $\beta$ and $c$):

1. KNN method to identify the best glob for each size.
2. Find the better fixed $\beta$s (better representatives of the “best glob” population)
3. Find for each better $\beta$ the best $c$ for each size. (peaks below the computational curve of a fixed $[\beta, c]$ couple have smaller $c$ and vice versa)
4. Find the best fixed $c$ for each better $\beta$ (the best representative of each “fixed $\beta$” population)
5. Find the best $[c, \beta]$ set considering the local curves (priority of fitting to the glob curves)
Conclusions (Hariri tests: global overview)

- Average fitting of the global size effect obtained by one single set with $c$ toward the smallest value.
- Detailed fitting of the global size effect varying $c$.
- Spread of the parameters sets to obtain the best fitting of the local size effect.
- Best individuals at borders!!!
- Structural effect on the model parameters.
- May parameters identification, solved as inverse problem, completely substitute investigation at micro or meso-scale?
KNN results (c and beta minimum)

FPZ width computed using the non-loc equ strain profile with a threshold of 20% of the peak (no exp cov)

\[ \text{fct} = 2.2 \times 10^5 \quad \text{nu} = 0.2 \]
\[ \text{eta} = 15.91 \quad \text{alpha} = 0.92 \]